Analyzing Mathematical Instructional Tasks

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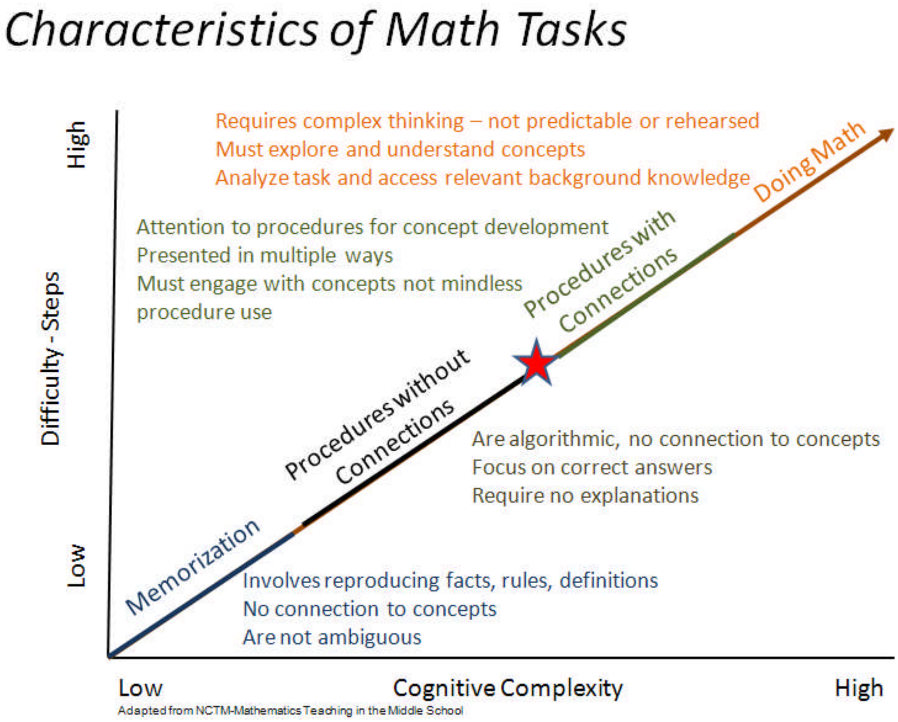
Mathematical tasks can be examined from a variety of perspectives including the number and kinds of representations evoked, the variety of ways in which they can be solved, and their requirements for student communication. In this book, we examine mathematical instructional tasks in terms of their cognitive demands. By cognitive demands we mean the kind and level of thinking required of students in order to successfully engage with and solve the task.

Here we describe a method for analyzing the cognitive demands of tasks as they appear in curricular or instructional materials. This focuses on tasks *before* the lesson begins, that is, the task as it appears in print form or as it is created by the teacher.

Why are the cognitive demands of tasks so important? It is important because opportunities for student learning are not created simply by putting students into groups, by placing manipulatives in front of them, or by handing them a calculator. Rather, it is the level and kind of thinking in which students engage that determines what they will learn. Tasks that require students to perform a memorized procedure in a routine manner lead to one type of opportunity for student thinking; tasks that demand engagement with concepts and that stimulate students to make purposeful connections to meaning or relevant mathematical ideas lead to a different set of opportunities for student thinking. Day-in and day-out, the cumulative effect of students' experiences with instructional tasks is students' implicit development of ideas about the nature of mathematics—about whether mathematics is something they personally can make sense of, and how long and how hard they should have to work to do so.

Since the tasks with which students become engaged in the classroom form the basis of their opportunities for learning mathematics, it is important to be clear about one's goals for student learning. Once learning goals for students have been clearly articulated, tasks can be selected or created to match these goals. Being aware of the cognitive demands of tasks is a central consideration in this matching. For example, if a teacher wants students to learn how to justify or explain their solution processes, she should select a task that is deep and rich enough to afford such opportunities. If, on the other hand, speed and fluency are the primary learning objectives, other types of tasks will be needed. In this chapter, readers will learn how to differentiate among the various levels of cognitive demand of tasks, thereby laying a foundation for more careful matching between the tasks teachers select for the classroom and their



goals for student learning.

DEFINING LEVELS OF COGNITIVE DEMAND OF MATHEMATICAL TASKS

Tasks with lower-level demands, for example, would consist of memorizing the equivalent forms of specific fractional quantities or performing conversions of fractions to percents or decimals using standard conversion algorithms in the absence of additional context or meaning (e.g., convert the fraction 3/8 to a decimal by dividing the numerator by the denominator to get .375; change .375 to a percent by moving the decimal point two places to the right to get 37.5%).

These lower-level tasks are classified as *memorization* and *procedures without connections to understanding*, *meaning, or concepts* (hereafter referred to simply as procedures without connections), respectively.

When tasks such as these are used, students typically work 10-30 similar problems within one sitting.

Another way in which students can be asked to think about the relationships among fractions, decimals, and percents—one that presents higher-level cognitive demands— might also use procedures, but do so in a way that builds connections to underlying concepts and meaning. For example students might be asked to use a 10 x 10 grid to illustrate how a fraction represents the same quantity as the decimal.

Students would also be asked to record their results on a chart containing the decimal, fraction, percent, and pictorial representations, thereby allowing them to make connections among the various representations and to attach meaning to their work by referring to the pictorial representation of the quantity every step of the way.

This task is classified as *procedures with connections to understanding, meaning, or concepts* (hereafter referred to simply as *procedures with connections).*

Another high-level task (classified as *doing mathematics)* would entail asking students to explore the relationships among the various ways of representing fractional quantities. Students would not—at least initially—be provided with the conventional conversion procedures. They might once again use grids, but this time grids of varying sizes (not just 10 x 10) would be used.

Students could be asked to shade six squares of a 4 x 10 rectangle and to represent the shaded area as a percent, a decimal, and a fraction. When students use the visual diagram to solve this problem, they are challenged to apply their understandings of the fraction, decimal, and percent concepts in novel ways. For example, once a student has shaded the six squares, he or she must determine how the six squares relate to the total number of squares in the rectangle.

Students typically perform far fewer problems sometimes as few as two or three in one sitting.